Equation (9) can be converted to give

$$V^2 = \frac{\gamma^0 RT}{M}\left[\frac{Z^2 U}{\gamma^0 SU - (\gamma^0 - 1) Y}\right]. \tag{10}$$

The quantity in brackets represents the correction factor for the ideal gas behaviour, since $\gamma^0 RT/M$ is the velocity value for an ideal gas. It can easily be verified that the quantity in the bracket reduces to unity at $P = 0$ (the state of the ideal gas).

It is also readily verified that equation (10) reduces to the following special case at very low pressures:

$$V^2 = \frac{\gamma^0 RT}{M}\left[1 + \frac{A}{RT}P\right], \tag{11}$$

where

$$A = B + (\gamma^0 - 1) T\frac{dB}{dT} + (\gamma^0 - 1)^2 \frac{T^2}{2\gamma^0}\frac{d^2 B}{dT^2}.$$

Equation (11) agrees with that derived by van Itterbeek *et al.* [4]. Hence equation (10) can be viewed as a generalization of van Itterbeek's to higher pressures.

It is to be noted that the quantities U, S and Y can readily be converted into functions expressed in terms of the virial coefficients and their derivatives.

Denoting

$$B_1 = T\frac{dB}{dT}, \qquad B_2 = T^2\frac{d^2 B}{dT^2},$$

$$C_1 = T\frac{dC}{dT}, \qquad C_2 = T^2\frac{d^2 C}{dT^2},$$

$$D_1 = T\frac{dD}{dT}, \qquad D_2 = T^2\frac{d^2 D}{dT^2}$$

and finding from equation (3b) the first and second derivatives of B', C' and D' in terms of the virial coefficients and their derivatives, it is then possible to show that

$$\left.\begin{aligned}
B' + T\frac{dB'}{dT} &= B_1/RT,\\
C' + T\frac{dC'}{dT} &= \{(C_1 - 2BB_1) - (C - B^2)\}/(RT)^2,\\
D' + T\frac{dD'}{dT} &= \{(D_1 - 3CB_1 - 3C_1 B - 6B^2 B_1) - 2(D - 3CB - 2B^3)\}/(RT)^3,
\end{aligned}\right\} \tag{12}$$

and

$$\left.\begin{aligned}
2T\frac{dB'}{dT} + T^2\frac{d^2 B'}{dT^2} &= B_2/RT,\\
2T\frac{dC'}{dT} + T^2\frac{d^2 C'}{dT^2} &= \{(C_2 - 2BB_2 - 2B_1^2) - 2[(C_1 - 2BB_1) - (C - B^2)]\}/(RT)^2,\\
2T\frac{dD'}{dT} + T^2\frac{d^2 D'}{dT^2} &= \{(D_2 - 3CB_2 - 3C_2 B - 6C_1 B_1 - 6B^2 B_2 - 12BB_1^2) -\\
&\quad - 4(D_1 - 3CB_1 - 3C_1 B - 6B^2 B_1) + 6(D - 3CB - 2B^3)\}/(RT)^3.
\end{aligned}\right\} \tag{13}$$

Substitution of equations (13), (3b) and (12) respectively into the expressions for U, S and Y will yield the required conversion.

3. CONCLUSION

Equation (10) presents no complications, when it is desired to determine the velocity of sound at a given pressure and temperature. Use of equation (1) requires a trial solution for ρ first. Also the present equation is a generalization of van Itterbeek's result [4] to higher pressures.

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